A Frequency Response Function-Based Damage Identification Method for Cylindrical Shell Structures

Usik Lee*, Wonhee Jeong, Jooyong Cho
Department of Mechanical Engineering, Inha University,
253 Yonghyun-Dong, Nam-Ku, Incheon 402-751, Korea

In this paper, a structural damage identification method (SDIM) is developed for cylindrical shells and the numerically simulated damage identification tests are conducted to study the feasibility of the proposed SDIM. The SDIM is derived from the frequency response function solved from the structural dynamic equations of damaged cylindrical shells. A damage distribution function is used to represent the distribution and magnitudes of the local damages within a cylindrical shell. In contrast with most existing modal parameters-based SDIMs which require the modal parameters measured in both intact and damaged states, the present SDIM requires only the FRF-data measured in the damaged state. By virtue of utilizing FRF-data, one is able to make the inverse problem of damage identification well-posed by choosing as many sets of excitation frequency and FRF measurement point as needed to obtain a sufficient number of equations.

Key Words : Structural Damage, Damage Identification, Cylindrical Shell, Frequency Response Function, Vibration

1. Introduction

The cylindrical shell is an important structural element that is not only applied for the civil and aerospace structures, but also for the mechanical, nuclear, chemical, and industrial structures. The oil or gas tanks, airplane fuselages, compressor shells, and boilers are the typical examples of application. Because such cylindrical shell structures should be free from structural failure due to structural damage, it is very important to understand the effects of structural damage on the structural behavior to detect the structural damage in the very early stage of damage progression. The dynamic characteristics of damaged cylin-

* Corresponding Author,
 E-mail : ulee@inha.ac.kr
 TEL : +82-32-860-7318; **FAX** : +82-32-866-1434
 Department of Mechanical Engineering, Inha University, 253 Yonghyun-Dong, Nam-Ku, Incheon 402-751, Korea. (Manuscript **Received** March 8, 2004; **Revised** September 13, 2004)

drical shells have been studied by some researchers including Roytman and Titova (2002).

Because the structural damages may change the vibration characteristics of a structure, the damage-induced changes in dynamic characteristics in turn can be used to detect and identify the structural damages. In most existing vibration-based structural damage identification methods (SDIMs), the modal parameters such as natural frequencies, modal damping and mode shapes and the frequency response function (FRF)-data have been widely used (Adams et al., 1978 ; Pandey et al., 1991 ; Banks et al. 1996 ; Choudhury and He, 1996 ; Wang et al., 1997 ; Bicanic and Chen, 1997 ; Thyagarajan et al., 1998 ; Hassiotis, 2000 ; Cho et al., 2002 ; Lee and Shin, 2002).

The SDIMs for cylindrical shells have been introduced by some researchers (Srinivasan and Kot, 1998 ; Royston et al., 2000 ; Ip and Tse, 2002). Srinivasan and Kot (1998) proposed to use the damage index method. The method is based on the changes in the modal strain energy of structures associated with damage and utilizes

the mode shape information. Royston et al. (2000) used the damage-induced slit-mode phenomena for detecting damages with in a nominally axisymmetric structure. Ip and Tse (2002) located the axial position of damage by using the sensitivities of natural frequencies to damage.

As discussed by many researchers (Bank et al., 1996 ; Choudhury and He, 1996 ; Wang et al., 1997 ; Thyagarajsn et al., 1998 ; Cho et al., 2002 ; Lee and Shin, 2002), there are apparent advantages of FRF-data over the modal parameters because (1) only a very limited number of modal parameters can be obtained by vibration tests because a large number of sensors can not be always applied for complete measurement, (2) the modal parameters can be contaminated by the modal extraction errors because they are extracted from the FRF data, and (3) the FRF-data can provide much more damage information in a desired frequency range than the modal parameters can do. In practice, it is easier to measure the FRF-data than to measure the modal data from a damaged structure that is in service. Thus, it seems to be very promising to use the measured FRF-data to detect and identify the damages generated within a structure.

Thus, the purposes of this paper are (1) to propose an FRF-data based SDIM for cylindrical shells and (2) to conduct the numerical feasibility tests for the proposed method. The method is derived from the frequency response function solved from governing differential equations of motion. In contrast with most existing modal parameters-based SDIMs, in which the modal parameters measured in both intact and damaged states are used, the present SDIM utilizes only the FRF-data measured in the damaged state.

2. Dynamics of Damaged Cylindrical Shells

2.1 Dynamic equations of motion

Consider an elastic, isotropic, thin intact cylindrical shell. The shell has the radius R, the length L, and the thickness h as shown in Fig. 1. The x-axis is directed along the symmetry axis of the median shell surface, the y-axis in the
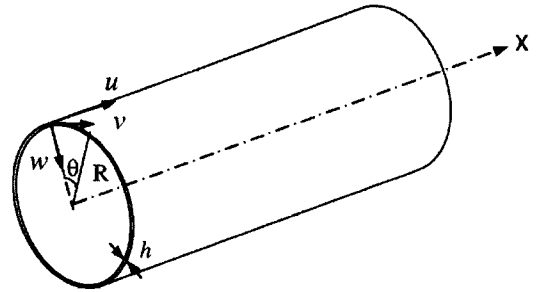

Fig. 1 Geometry of a cylindrical shell

circumferential direction, and the z-direction along the interior normal of the meridian surface. Define the displacements in the longitudinal, circumferential and radial directions by $u(x, \theta, t)$, $v(x, \theta, t)$ and $w(x, \theta, t)$, respectively, and also define the external loads in each direction by $p_x(x, \theta, t)$, $p_y(x, \theta, t)$ and $p_z(x, \theta, t)$, respectively.

The Donnell-Mushitari's thin shell theory gives the dynamic equations of motion for the cylindrical shell as (Markus, 1988 ; Soedel, 1993)

$$[L]\{u(x, \theta, t)\}+\{f(x, \theta, t)\} = \rho h\{\ddot{u}(x, \theta, t)\} \tag{1}$$

where $\{u(x, \theta, t)\}$ and $\{f(x, \theta, t)\}$ are the displacements field vector and the external loads vector, respectively, defined by

$$\{u(x, \theta, t)\}=\{u(x, \theta, t)\ v(x, \theta, t)\ w(x, \theta, t)\}^{\mathrm{T}}$$
$$\{f(x, \theta, t)\}=\{p_x(x, \theta, t)\ p_y(x, \theta, t)\ p_z(x, \theta, t)\}^{\mathrm{T}} \tag{2}$$

and $[L]$ is the matrix of differential operators for intact cylindrical shells defined by

$$[L]=K\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \tag{3}$$

where

$$L_{11}=\frac{\partial^2}{\partial x^2}+\frac{(1-\nu)}{2R^2}\frac{\partial^2}{\partial \theta^2}$$

$$L_{12}=L_{21}=\frac{(1+\nu)}{2R}\frac{\partial^2}{\partial x\partial\theta}$$

$$L_{13}=-L_{31}=-\frac{\nu}{R}\frac{\partial}{\partial x} \tag{4}$$

$$L_{22} = \frac{(1-\nu)}{2} \frac{\partial^2}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}$$

$$L_{23} = -L_{32} = -\frac{1}{R^2} \frac{\partial}{\partial \theta}$$

$$L_{33} = -\frac{h^2}{12} \nabla^4 - \frac{1}{R^2}$$

In above equations, the dot $(\,\dot{}\,)$ indicates the derivative with respect to the time t and ρh is the mass density per unit area, ν is the Poisson ratio, and K is the membrane rigidity of shell defined by (Soedel, 1993)

$$K = \frac{Eh}{1-\nu^2} \qquad (5)$$

In most practical cases, it is impossible to assign definitive representations of the stiffnesses at local damage sites because the locations, dimensions, and geometries of local damages are *not* known in prior. Thus, one of the simplest approaches is to represent the damage–induced change in stiffness by the degradation of elastic modulus as follows (Banks et al., 1996)

$$E_d(x, \theta) = E[1 - d(x, \theta)] \qquad (6)$$

where E is Young's modulus in the intact state and E_d is the effective Young's modulus in the damaged state, and $d(x, \theta)$ is the damage distribution function by which the distribution and states of local damages (i.e., damage locations, sizes, and severities) can be characterized. For instance, $d(x, \theta) = 0$ indicates the intact state, while $d(x, \theta) = 1$ indicates the complete ruptures of material at (x, θ) due to damage. In general, the damage–induced decrease in the mass density is neglected because the damage will not result in complete breakage with the loss of mass (Nicanic and Chen, 1997 ; Hassiotis, 2000).

Assume that the local damages within a cylindrical shell are uniform through the thickness, (*i.e.*, thickness –through damages). The dynamic equations of motion for damaged cylindrical shells can be obtained from the Donnell–Mushitari's thin shell theory by replacing the intact Young's modulus E with the effective Young's modulus E_d as follows :

$$[\bar{L}]\{u(x, \theta, t)\} + \{f(x, \theta, t)\} = \rho h \{\ddot{u}(x, \theta, t)\} \qquad (7)$$

where $[\bar{L}] = [L] + [L_D]$ is the matrix of differential operators for damaged cylindrical shells and $[L_D]$ is defined by

$$[L_D] = -K \begin{bmatrix} L_{D11} & L_{D12} & L_{D13} \\ L_{D21} & L_{D22} & L_{D23} \\ L_{D31} & L_{D32} & L_{D33} \end{bmatrix} \qquad (8)$$

where

$$L_{D11} = \frac{\partial}{\partial x}\left(d \frac{\partial}{\partial x}\right) + \frac{2(1-\nu)}{R^2} \frac{\partial}{\partial \theta}\left(d \frac{\partial}{\partial \theta}\right)$$

$$L_{D12} = \frac{\nu}{R} \frac{\partial}{\partial x}\left(d \frac{\partial}{\partial \theta}\right) + \frac{2(1-\nu)}{R} \frac{\partial}{\partial \theta}\left(d \frac{\partial}{\partial x}\right)$$

$$L_{D13} = -\frac{1}{R}\left(\frac{\partial d}{\partial x} + d \frac{\partial}{\partial x}\right)$$

$$L_{D21} = \frac{\nu}{R} \frac{\partial}{\partial \theta}\left(d \frac{\partial}{\partial x}\right) + \frac{2(1-\nu)}{R} \frac{\partial}{\partial x}\left(d \frac{\partial}{\partial \theta}\right)$$

$$L_{D22} = 2(1-\nu) \frac{\partial}{\partial x}\left(d \frac{\partial}{\partial x}\right) + \frac{1}{R^2} \frac{\partial}{\partial \theta}\left(d \frac{\partial}{\partial \theta}\right) \qquad (9)$$

$$L_{D23} = -\frac{1}{R^2}\left(\frac{\partial d}{\partial \theta} + d \frac{\partial}{\partial \theta}\right)$$

$$L_{D31} = \frac{\nu}{R}\left(d \frac{\partial}{\partial x}\right), \quad L_{D32} = \frac{1}{R^2}\left(d \frac{\partial}{\partial \theta}\right)$$

$$\begin{aligned} L_{D33} = &-\frac{h^2}{12}\left[\frac{\partial^2}{\partial x^2}\left\{d\left(\frac{\partial^2}{\partial x^2} + \frac{\nu}{R^2} \frac{\partial^2}{\partial \theta^2}\right)\right\}\right. \\ &+ \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}\left\{d\left(\nu\frac{\partial^2}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2}{\partial \theta^2}\right)\right\} \\ &\left. + \frac{2(1-\nu)}{R^2} \frac{\partial^2}{\partial x \partial \theta}\left(d \frac{\partial^2}{\partial x \partial \theta}\right)\right] - \frac{d}{R^2} \end{aligned}$$

For the intact case, $d(x, \theta) = 0$ and the matrix of differential operators $[L_D]$ vanishes to bring Eq. (7) back to Eq. (1).

2.2 Forced vibration responses

Assume that only the external load $p_z(x, \theta, t)$ which is normal to shell surface is harmonically applied at a specified point (x_F, θ_F) as follows :

$$p_z(x, \theta, t) = F_o \delta(x - x_F) \delta(\theta - \theta_F) e^{i\omega t} \qquad (10)$$

where F_o is the amplitude of the harmonic point force and ω is the excitation circular frequency. The forced vibration response of a damaged cylindrical shell can be assumed in the form

$$\{\boldsymbol{u}(x,\ \theta,\ t)\}=\sum_{I=1}\{\boldsymbol{U}_I(x,\ \theta)\}q_I(t) \qquad (11)$$

where $\{\boldsymbol{U}_I\}=\{U_I\ V_I\ W_I\}^{\mathrm{T}}$ are the normal modes of the intact cylindrical shell and q_I are the modal coordinates. The normal modes $\{\boldsymbol{U}_I\}$ satisfy the eigenvalue problem of the intact cylindrical shell

$$[\boldsymbol{L}]\{\boldsymbol{U}_I\}=-\rho h\Omega_I^2\{\boldsymbol{U}_I\} \quad (no\ sum\ on\ I) \quad (12)$$

and the orthogonality property

$$\int_A \rho h\{\boldsymbol{U}_I\}^T\{\boldsymbol{U}_J\}dxd\theta=\delta_{IJ} \qquad (13)$$

$$\int_A \{\boldsymbol{U}_I\}^T[\boldsymbol{L}]\{\boldsymbol{U}_J\}dxd\theta \\ =-\Omega_I^2\delta_{IJ}(no\ sum\ on\ I) \qquad (14)$$

where Ω_I are the natural frequencies of the intact shell and δ_{IJ} is the Kronecker symbol.

Substituting Eq. (11) into Eq. (7) and applying Eqs. (12-14) yields a set of coupled modal equations

$$\ddot{q}_I+\Omega_I^2q_I-\sum_{J=1}\lambda_{IJ}q_J=f_I(t) \quad (no\ sum\ on\ I) \quad (15)$$

where f_I are the modal forces given by

$$f_I(t)=\int_0^L\int_0^{2\pi}p_z(x,\ \theta,\ t)\ W_I dxd\theta \\ =W_I(x_F,\ \theta_F)F_oe^{i\omega t} \qquad (16)$$

and λ_{IJ} is the symmetric matrix defined by

$$\lambda_{IJ}=\int_A\{\boldsymbol{U}_I\}^T[\boldsymbol{L}_D]\{\boldsymbol{U}_J\}dxd\theta \qquad (17)$$

The matrix λ_{IJ} is the damage influence matrix (DIM). Equation (15) implies that, if DIM is not a diagonal matrix, the damage may induce the coupling between modal coordinates. The natural frequencies of a damaged shell can be obtained from

$$\det\lfloor(\Omega_I^2-\bar{\Omega}_I^2)\delta_{IJ}-\lambda_{IJ}\rfloor=0 \quad (no\ sum\ on\ I) \quad (18)$$

where $\bar{\Omega}_I$ are the natural frequencies of damaged shell.

For a cylindrical shell simply-supported at both ends, the normal modes $\{\boldsymbol{U}_I\}$ are given by (Soedel, 1993)

$$\{\boldsymbol{U}_I\}=\begin{Bmatrix}U_I\\V_I\\W_I\end{Bmatrix}\equiv\begin{Bmatrix}U_{mni}\\V_{mni}\\W_{mni}\end{Bmatrix}$$

$$=\begin{Bmatrix}A_{mni}\cos\left(\dfrac{m\pi x}{L}\right)\cos(n\theta)\\[2mm] B_{mni}\sin\left(\dfrac{m\pi x}{L}\right)\sin(n\theta)\\[2mm] C_{mni}\sin\left(\dfrac{m\pi x}{L}\right)\cos(n\theta)\end{Bmatrix} \qquad (19)$$

where $i=1,\ 2,\ 3$. Equation (19) implies that there are three natural frequencies for every m, n combination. The lowest is associated with the mode where the transverse component dominates, while the other two are usually higher by an order of magnitude and are associated with the mode where the displacements in the tangent plane dominate. Because U_I and V_I can be expressed in terms of W_I, Eq. (19) can be rewritten as

$$\{\boldsymbol{U}_I\}=\left\{P_{1I}\dfrac{\partial W_I}{\partial x}\ \ P_{2I}\dfrac{\partial W_I}{\partial\theta}\ \ W_I\right\}^T \qquad (20)$$

where

$$P_{1I}=\dfrac{L}{m\pi}\dfrac{A_{mni}}{C_{mni}},\ P_{2I}=-\dfrac{1}{n}\dfrac{B_{mni}}{C_{mni}} \qquad (21)$$

As used in above equations, the contracted subscripts will be consistently used for the mode numbers in the following for brevity, i.e., I for mni and J for rsj.

By substituting Eq. (20) into Eq. (17) and by integrating by parts, one may obtain.

$$\lambda_{IJ}=\int_A d(x,\ \theta)\Phi_{IJ}(x,\ \theta)\ dxd\theta \qquad (22)$$

with

$$\Phi_{IJ}(x,\ \theta)=Q_{1IJ}W_IW_J+Q_{2IJ}\dfrac{\partial^2 W_I}{\partial x\partial\theta}\dfrac{\partial^2 W_J}{\partial x\partial\theta} \quad (23)$$

where

$$Q_{1IJ} = \left(P_{1I}P_{1J} + \frac{h^2}{12}\right)\left(\frac{m\pi}{L}\right)^2\left(\frac{r\pi}{L}\right)^2 + (P_{2I}P_{2J} + P_{2I} + P_{2J} + 1)\frac{h^2}{12}\frac{1}{R^4}n^2s^2$$
$$+ \left\{P_{1I}P_{1J}\frac{\nu}{R} + (P_{2J}+1)\frac{h^2}{12}\frac{\nu}{R^2}\right\}\left(\frac{m\pi}{L}\right)^2 s^2$$
$$+ \left\{P_{1I}P_{1J}\frac{\nu}{R} + (P_{2I}+1)\frac{h^2}{12}\frac{\nu}{R^2}\right\}n^2\left(\frac{r\pi}{L}\right)^2 \quad (24)$$
$$+ \frac{1}{R^2}(P_{2I}n^2 + P_{2J}s^2 + 1) + P_{2I}P_{2J}\frac{1}{R^2}n^2s^2 + \frac{\nu}{R}\left(\frac{\pi}{L}\right)^2(P_{1I}m^2 + P_{1J}r^2)$$

$$Q_{2IJ} = (1-\nu)\left\{P_{1I}P_{1J}\frac{1}{2R^2} + (P_{1I}P_{2J} + P_{2I}P_{1J})\frac{1}{2R} + \frac{P_{2I}P_{2J}}{2}\right.$$
$$\left. + (P_{2I}P_{2J} + 2P_{2I} + 2P_{2J} + 2)\frac{h^2}{12}\frac{1}{R^2}\right\}$$

Equation (22) shows that DIM depends on the distribution pattern of damage over the shell. Once the damage distribution function $d(x, \theta)$ is given, DIM can be readily computed from Eq. (22).

Assume there are N local damages of constant magnitudes $0 \le D_k \le 1$ $(k=1, 2, \cdots, N)$. They are centered at (x_{dk}, θ_{dk}) and uniformly distributed over a small finite segment of area $2\bar{x}_k \times 2R\bar{\theta}_k$, as shown in Fig. 2. The damage distribution function can be then written as

$$d(x, \theta) = \sum_{k=1}^{N} D_k[H(x_{dk} - \bar{x}_k) - H(x_{dk} + \bar{x}_k)] \quad (25)$$
$$\times [H(\bar{\theta}_{dk} - \bar{\theta}_k) - H(\bar{\theta}_{dk} + \bar{\theta}_k)]$$

where $H(y-a)$ is the Heaviside's unit function defined by

$$H(y-a) = \begin{cases} 1 \text{ when } y > a \\ 0 \text{ when } y < a \end{cases} \quad (26)$$
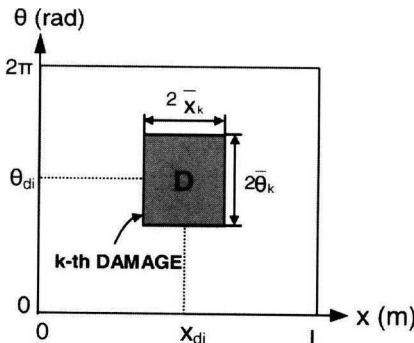
Substituting Eq. (24) into Eq. (22) yields

Fig. 2 A typical local damage in a cylindrical shell

$$\lambda_{IJ} = \sum_{k=1}^{N} k_{IJ}^k D_k \quad (27)$$

where

$$k_{IJ}^k = \int_{\theta_{dk} - \bar{\theta}_k}^{\theta_{dk} + \bar{\theta}_k} \int_{x_{dk} - \bar{x}_k}^{x_{dk} + \bar{x}_k} \Phi_{IJ}(x, \theta) \, dx \, d\theta \quad (28)$$

Solving Eq. (15) for q_I and substituting the results into Eq. (11) may yield the forced vibration responses of damaged cylindrical shell in the form:

$$\{u(x, \theta, t)\} = \left(\sum_{I=1} \{U_I\}\frac{W_I(x_F, \theta_F)}{\Omega_I^2 - \omega^2}\right.$$
$$\left. + \sum_{I=1}\sum_{J=1}\lambda_{IJ}\{U_I\}\frac{W_J(x_F, \theta_F)}{(\Omega_I^2 - \omega^2)(\Omega_J^2 - \omega^2)}\right)F_0 e^{i\omega t}$$
$$= \{U(x, \theta) \quad V(x, \theta) \quad W(x, \theta)\}^T F_0 e^{i\omega t}$$
$$(29)$$

The effects of structural damping can be readily taken into account in Eq. (29), by simply replacing the natural frequencies Ω_I with $\Omega_I(1 + i\eta_I)^{1/2}$, where η_I is the modal loss factor.

3. Damage Identification Method

It is in general easier to measure the radial displacement $w(x, \theta, t)$ rather than to measure the longitudinal displacement $u(x, \theta, t)$ or circumferential displacement $v(x, \theta, t)$. Thus, the inertance FRF of $w(x, \theta, t)$ measured from a damaged cylindrical shell will be considered as the experimentally measured data required to identify the damages within a cylindrical shell.

The inertance FRF of the radial displacement $w(x, \theta, t)$ is defined as the ratio of the acceleration measured at a point (x, θ) to the force applied at (x_F, θ_F) as

$$A(\omega; x, \theta) = \frac{\ddot{w}(x, \theta, t)}{p_z(x_F, \theta_F, t)} \quad (30)$$
$$= -\omega^2 W(x, \theta)$$

where $\ddot{w}(x, \theta, t)$ and $p_z(x, \theta, t)$ denote the measured acceleration and the point force applied normal to the shell surface, respectively. Applying $p_z(x, \theta, t)$ from Eq. (10) and $w(x, \theta, t)$ from Eqs. (29) into Eq. (30) yields

$$-\omega^2 \sum_{k=1}^{N} \left(\sum_{I=1}^{3M} \sum_{J=1}^{3M} k_{IJ}^k \frac{W_I(x, \theta)}{\Omega_I^2 - \omega^2} \frac{W_K(x_F, \theta_F)}{\Omega_J^2 - \omega^2} \right) D_k$$
$$= A(\omega ; x, \theta) + \omega^2 \sum_{I=1}^{3M} \frac{W_I(x, \theta) W_I(x_F, \theta_F)}{\Omega_I^2 - \omega^2} \quad (31)$$

Equation (31) certainly provides the relationship between *unknown* damage information (*i.e.*, damage locations k and magnitudes D_k) and *known* vibration data such as the inertance FRF $A(\omega ; x, \theta)$, natural modes W_I, and natural frequencies Ω_I, it can be used to develop an algorithm for structural damage identification. In Eq. (31), the mode shapes and natural frequencies in the intact state are considered as the *known* quantities because they will be provided in advance by modal testing or analytical vibration analysis. The inertance FRF $A(\omega ; x, \theta)$ is also considered as the *known* quantity because it will be measured directly from the damaged cylindrical shell. However, the damage magnitudes D_k ($k=1, 2, \cdots, N$) are *unknowns* to be identified.

In Eq. (31), the FRF-measurement point (x, θ) and excitation frequency ω can be chosen arbitrary. For a chosen set of (x, θ) and ω, Eq. (31) may yield a linear algebraic equation for unknown D_k ($k=1, 2, \cdots, N$). Thus, by choosing N sets of excitation frequency and FRF-measurement point, total N linear algebraic equations can be derived from Eq. (31) in the form

$$[X]\{D\} = \{Y\} \quad (32)$$

where

$$\{D\} = \{D_1 \ D_2 \ D_3 \ \cdots \ D_N\}^{\mathrm{T}} \quad (33)$$

$$\{Y\} = \{Y_1 \ Y_2 \ Y_3 \ \cdots \ Y_N\}^{\mathrm{T}} \quad (34)$$

$$[X] = [X_{kl}] \quad (35)$$

with

$$X_{kl} = -\omega_q^2 \sum_{I=1}^{3M} \sum_{J=1}^{3M} k_{IJ}^l \frac{W_I(x_p, \theta_P)}{\Omega_I^2 - \omega_q^2} \frac{W_J(x_F, \theta_F)}{\Omega_J^2 - \omega_q^2} \quad (36)$$

$$Y_k = A(\omega_q ; x_p, \theta_p)$$
$$+ \omega_q^2 \sum_{I=1}^{3M} \frac{W_I(x_p, \theta_p) W_I(x_F, \theta_F)}{\Omega_I^2 - \omega_q^2} \quad (37)$$

$$k_{IJ}^l = \int_{\theta_{dl} - \theta_l}^{\theta_{dl} + \theta_l} \int_{x_{dl} - x_l}^{x_{dl} + x_l} \Phi_{IJ}(x, \theta) \, dx \, d\theta \quad (38)$$

$$k = p + (q-1)P \text{ and } l = 1, 2, \cdots, N$$
$$(p = 1, 2, \cdots, P \text{ and } q = 1, 2, \cdots, Q) \quad (39)$$

In the above equations, (x_p, θ_p) and ω_q denote chosen FRF-measurement points and excitation frequencies, respectively. Once the matrix $[X]$ and vector $\{Y\}$ are computed by using the known vibration data (i.e., inertance FRF $A(\omega ; x, \theta)$, natural modes W_I, and natural frequencies Ω_I), Eq. (32) can be solved for $\{D\}$. Because the subscript k for D_k indicates the location of the k-th finite segment, the value of D_k itself indicates the damage magnitude uniform over the k-th finite segment. Thus, solving Eq. (32) for $\{D\}$ may imply the identification of all local damages at a time.

By virtue of using FRF-data, one may choose as many sets of excitation frequency and FRF-measurement point as needed to obtain a square matrix $[X]$. For example, for a cylindrical shell divided into N finite segments with unknown damage magnitudes D_k ($k=1, 2, \cdots, N$), one may determine the number of FRF-measurement points P and the number of excitation frequencies Q to satisfy the relation $PQ = N$. This can make the inverse problem represented by the N by N matrix $[X]$ well-posed, and a direct inverse method can be applied to Eq. (32), instead of using a complicated optimization solution technique. This will be one of advantages of the present SDIM.

In summary, Eq. (32) represents the structural damage identification algorithm which is applicable to cylindrical shells, and it requires : (1) the natural frequencies measured in the intact state (*i.e.*, Ω_I), (2) the natural modes measured in the intact state (*i.e.*, W_I), and (3) the FRF-data measured in the damaged state, *i.e.*, $A(\omega ; x, \theta)$.

4. Numerical Illustrations and Discussions

To investigate the effects of damage on the natural frequencies and also to test the feasibility of the present SDIM, a cylindrical shell sim-

ply supported at both end edges is considered. The cylindrical shell has the radius $R=0.125\ m$, length $L=0.3\ m$, thickness $h=0.003\ m$, Young's Modulus $E=206\ GPa$, and the mass density $\rho=7850\ kg/m^3$. As shown in Fig. 3, two cases are considered : the first case is when the shell has one local damage $D=0.3$ (Fig. 3(a)), and the second case is when the shell has three local damages $D_1=0.3$, $D_2=0.4$ and $D_3=0.2$ (Fig. 3

(a) One damage   (b) Three damages

Fig. 3 A simply supported cylindrical shell with one damage (a) and three damages (b)

(b)). To compute the inertance FRF required for structural damage identification, the cylindrical shell is excited by applying a harmonic point force at the point $(x_F=0.15\ m,\ \theta_F=\pi)$, which is indicated by the cross $(\times)$ in Fig. 3.

4.1 Effect of damage on the natural frequencies

Figure 4 compares the natural frequencies of the intact shell with those of the damaged shells with one and three damages, with varying the circumferential mode number n for longitudinal mode numbers $m=1$, 2, 3, and 4. A sufficient number of normal modes (i.e., $m\times n=10\times11$ modes) are used to compute the natural frequencies. All natural frequencies are found to slightly decrease in magnitude due to the presence of local damages. As shown for the intact shell in the book by Soedel (1993), Fig. 4 certainly shows that the natural frequencies of damaged
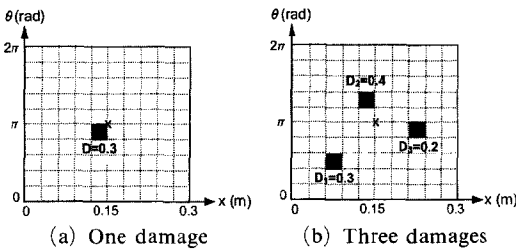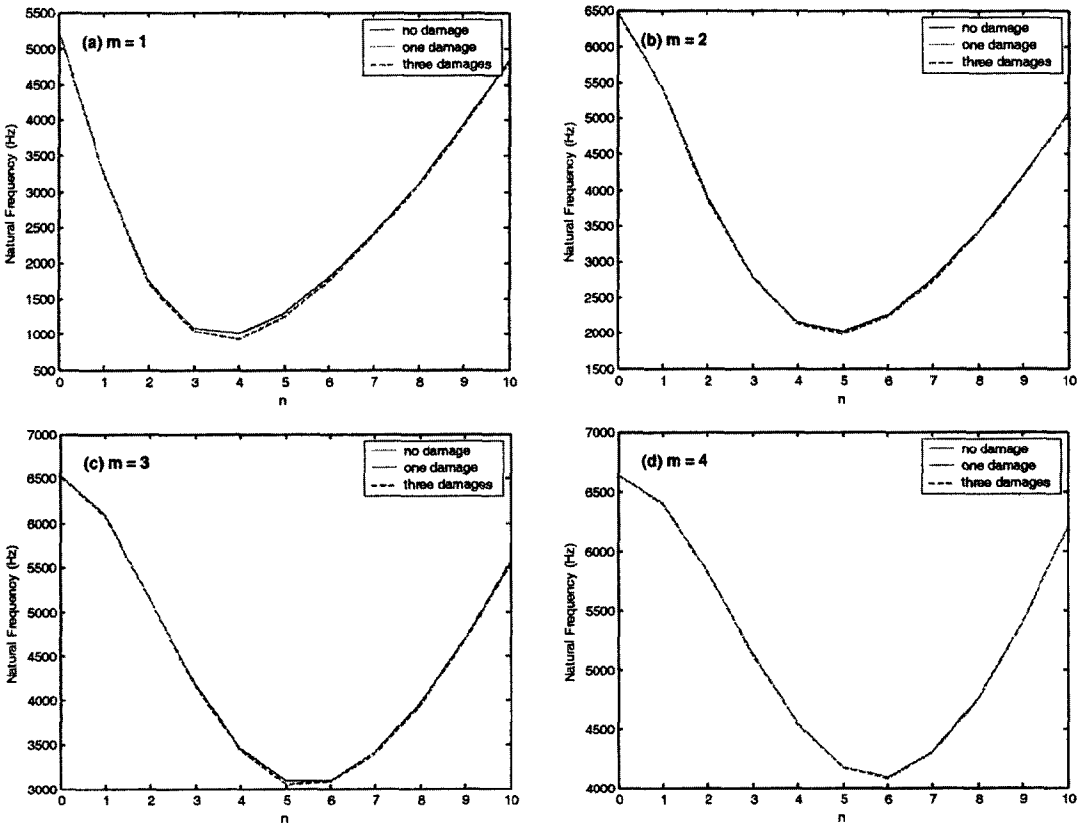

Fig. 4 The effect of damage on the natural frequencies

shell gradually decrease in magnitude to reach the minimum values as the circumferential mode number n is increased, for fixed longitudinal mode number m, and then continuously increase as the circumferential mode number n is kept increasing.

4.2 Numerical damage identification tests

The present SDIM requires the inertance FRF in the damaged state and the modal data (*i.e.*, natural modes and natural frequencies) in the intact state. For the present numerical damage identification tests, the required modal data are analytically obtained from the eigenvalue problem of Eq. (12) by assuming that Eq. (12) represents the *refined* cylindrical shell model in the intact state. By the word 'refined', we mean

that the experimentally measured and analytically predicted modal parameters are in good agreement.

Because the inertance FRF data is experimentally measured directly from the damaged shell, it is apt to be contaminated by the measurement noises. Thus, following the approach used by Thyagarajan et al.(1998), an $e\%$ random noise is added to the inertance FRF analytically obtained from Eq. (30) to take into account any possible noises in the experimentally measured inertance FRF. That is

$$\bar{A}(\omega_q \; ; \; x_p, \; \theta_p)$$
$$= A(\omega_q \; ; \; x_p, \; \theta_p)\left(1 + \frac{e}{100} \times randn\right) \quad (40)$$

where $\bar{A}$ is the inertance FRF contaminated by

(a) 2×2 elements

(b) 4×4 elements

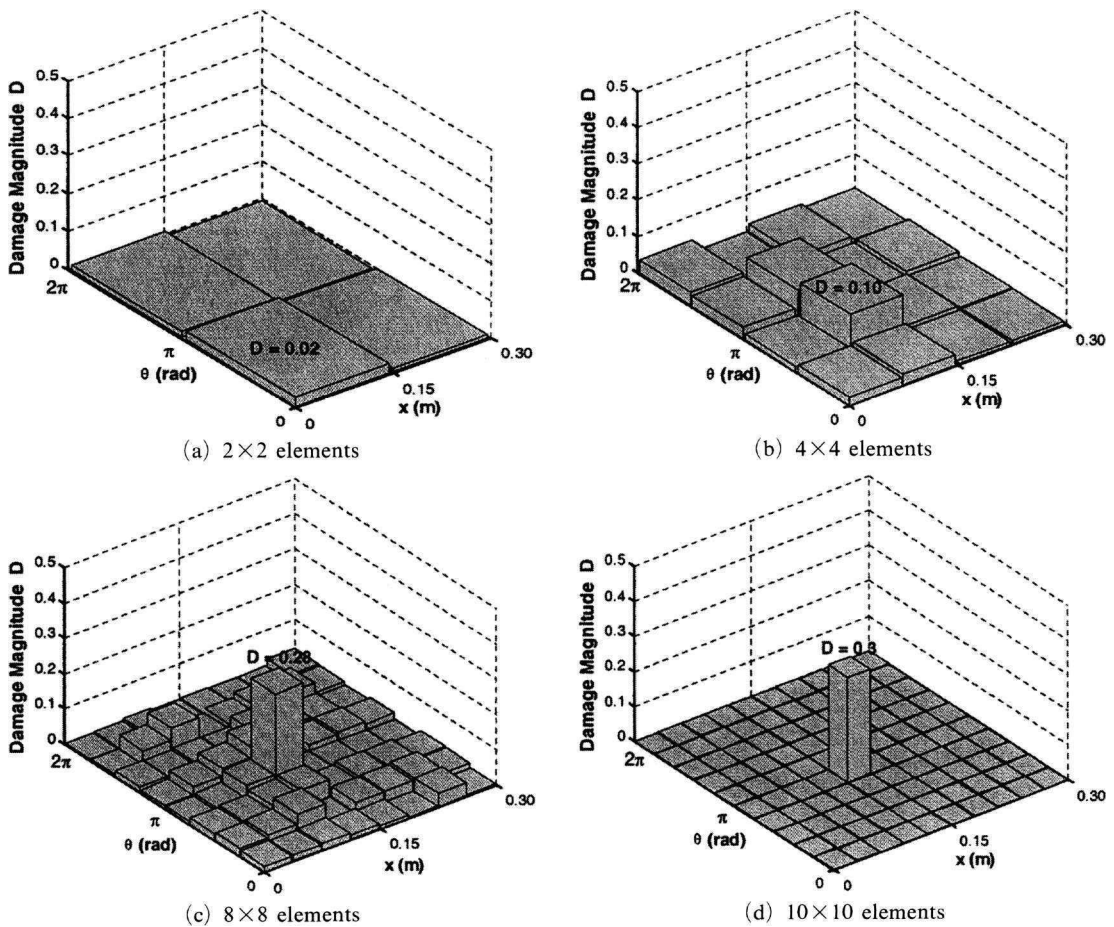(c) 8×8 elements

(d) 10×10 elements

Fig. 5 Damage dentification results vs. the number of finite segments used in analysis

e% random noises, and *randn* is the random noise generator function in MATLAB®. It is assumed that the random noise is uniformly distributed, with its mean value is zero and the variance is equal to one. In the previous study (Cho *et al.*, 2002), the numerical feasibility test based on Eq. (40) was proved to be quite reasonable approach through the experiments for damaged beam-type structures.

Figure 5 compares the damage identification results for the one damage problem obtained by using different number of finite shell segments, considering 3% random noises in FRF-data. Figure 5 clearly shows that the accuracy of damage identification is improved as the total number of finite shell segments is increased. Using

about 10×10 elements is found to give almost converged, accurate results.

Figure 6 compares the damage identification results for the three damages problem obtained by considering different levels of random noise. As expected, the accuracy of damage identification results is shown to be degraded as the level of random noise is increased. However, Fig. 6 shows that the present SDIM provides very satisfactory damage identification results up to quite high level of random noise, say 7%.

5. Conclusions

In this paper, an FRF-based SDIM is proposed for cylindrical shell structures and the

(a) 0% random noise

(b) 3% random noise

(c) 5% random noise
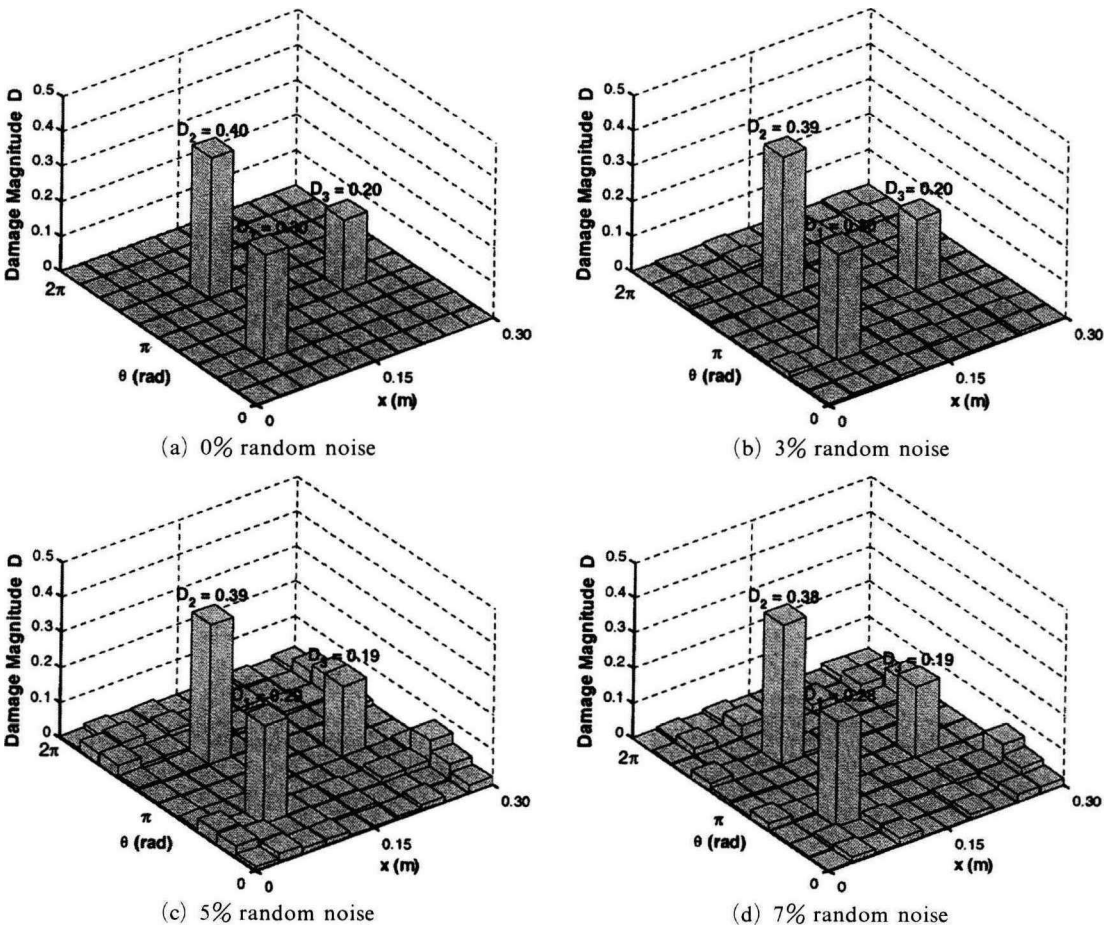
(d) 7% random noise

Fig. 6 Damage identification results for the three damages problem obtained by varying the level of random noises in FRF-data

numerically simulated damage identification tests are conducted to evaluate the performance of the present SDIM. The structural damage identification algorithm is derived from the frequency response function solved from the dynamic equations of motion of the damaged shell, in which local damages are characterized by using a damage distribution function.

The appealing features of the present SDIM are as follows.

(1) The FRF–data, instead of modal data, is required for the damaged state.

(2) To form a well-posed damage identification problem, a sufficient number of linear algebraic equations can be readily obtained by simply varying the combination of excitation frequency and FRF-measurement point.

(3) Many local damages can be located and quantified simultaneously.

For the successful application of the proposed SDIM, the errors in both the modal data (measured from the intact structure) and the FRF-data (measured from damaged structure) must be small enough, especially for successful identification of small and weak local damages.

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